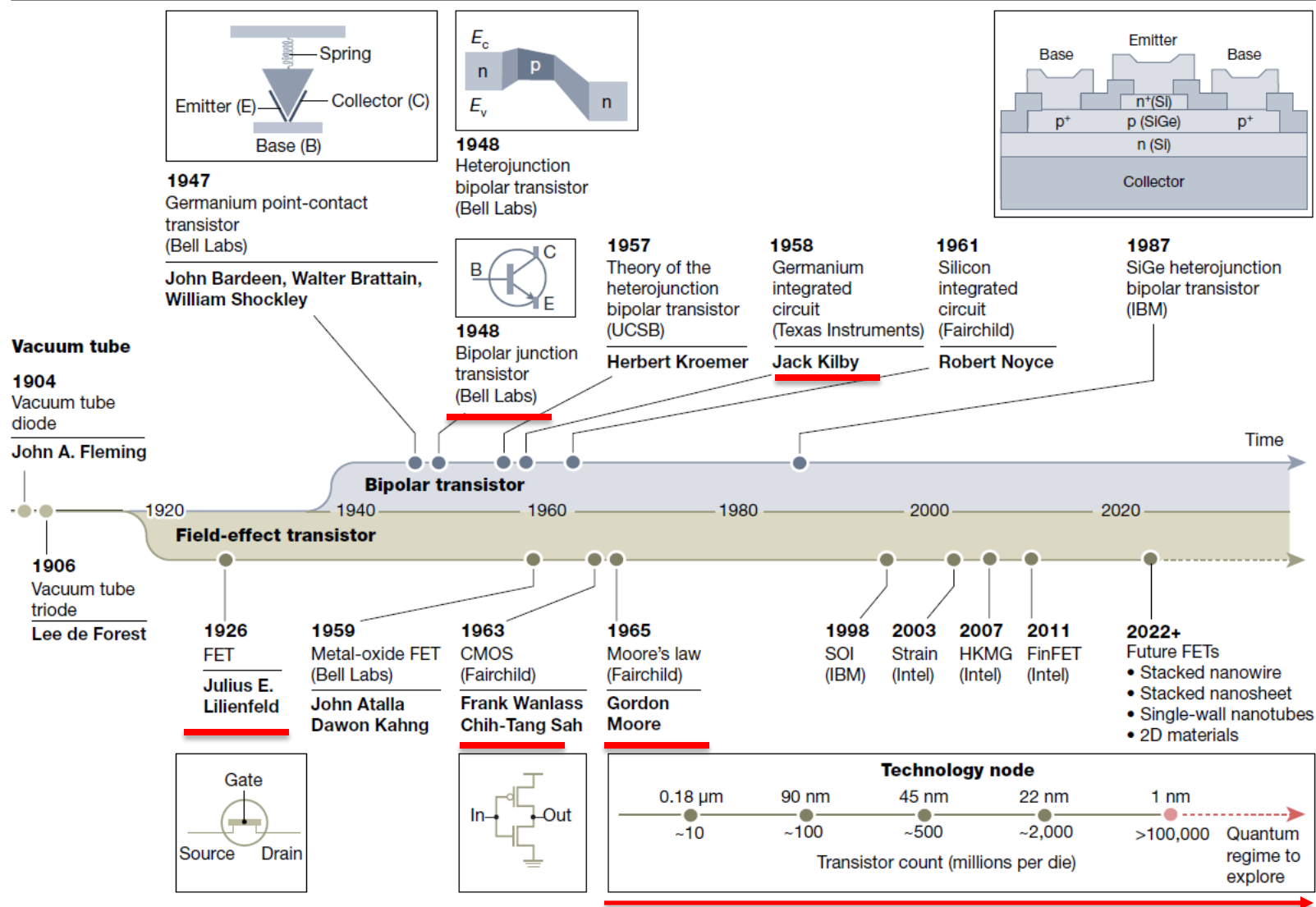


# CHAPTER 1

## Signals and Amplifiers

# Perspective



**Fig. 1 | The history of transistor technology.** All major transitions from vacuum tubes to BJTs, and eventually to MOSFETs, have been primarily driven by the need to reduce power consumption. Four major non-traditional FET scaling technologies, that is, SOI, strained channel, HKMG and FinFET are shown according to their commercialization time, corresponding to the 0.18-μm, 90-nm, 45-nm and 22-nm technology nodes, respectively. It is noted

that beginning from the 22-nm node, the technology node becomes increasingly smaller than the FET physical dimension.  $E_c$ , conduction band minima;  $E_v$ , valence band maxima; IBM, International Business Machines; UCSB, University of California, Santa Barbara. Transistor count data are from [https://en.wikipedia.org/wiki/Transistor\\_count](https://en.wikipedia.org/wiki/Transistor_count).

Cao W, Bu H, Vinet M, Cao M, Takagi S, Hwang S, Ghani T, Banerjee K. The future transistors. Nature. 2023 Aug; 620(7974):501-515. doi: 10.1038/s41586-023-06145-x. Epub 2023 Aug 16. PMID: 37587295.

# Outlines

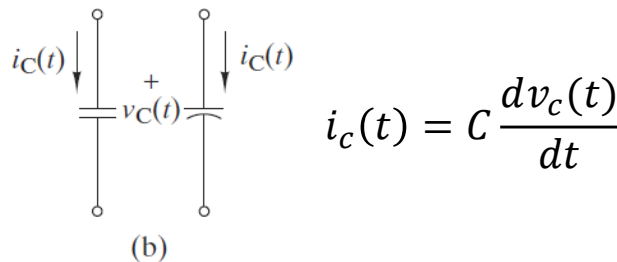
- 1.1 Signals and Overview
- 1.2 Amplifiers
- 1.3 Circuit Models for Amplifiers
- 1.4 Frequency Response of Amplifier

# 1.1 Signals

- Resistance ( $R, \Omega$ ):  $Z_R = R$
- Capacitance **RT PP.274**
  - C, F-Farad
  - Resistance:  $Z_C = \frac{1}{j\omega C}$ ,  $\omega = 2\pi f$ 

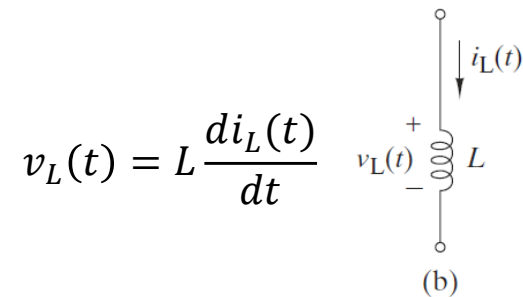
Angular frequency  
in radians/s

Frequency  
in Hz
- Inductance **RT PP.282**
  - L, H-Henry
  - Resistance:  $Z_L = j\omega L$ ,  $\omega = 2\pi f$



**FIGURE 6-1** The capacitor:  
(a) Parallel plate device.  
(b) Circuit symbols.

Linear circuit textbook:  
Roland E Thomas (RT), e.g.  
the analysis and design of  
linear circuit, 8<sup>th</sup> edition



**FIGURE 6-9** (a) Magnetic flux surrounding a current-carrying coil. (b) Circuit symbol showing inductor current and voltage.

- Things will be used from linear circuit class
  - KCL, KVL: [RT PP.23](#)
  - Equivalent circuit: [RT PP.41, Fig. 2-37](#)
  - Voltage and current division: [RT PP.42](#)
  - Thévenin and Norton Equivalent Circuits: [RT PP.107](#)
  - ... Catch up the 1~4 chapters

- Kirchhoff's Voltage Law (KVL)
  - The **algebraic** sum of all the voltages around a loop is zero at every instant.
- Kirchhoff's Current Law (KCL)
  - The sum of the currents entering a node equals the sum of the currents leaving the node.
- RT PP. 23

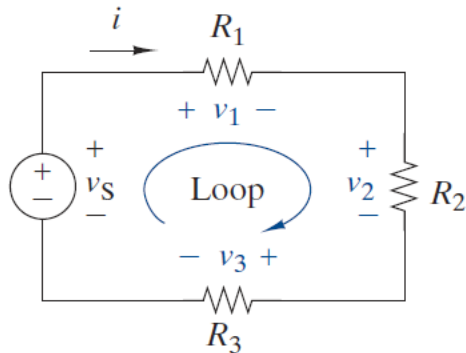


FIGURE 2-38 A voltage divider circuit.

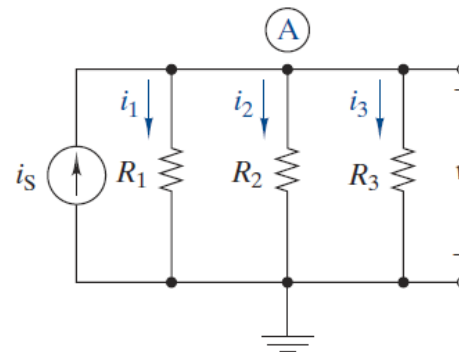


FIGURE 2-44 A current divider circuit.

• Equivalent circuit **RT PP.41, Fig. 2-37**

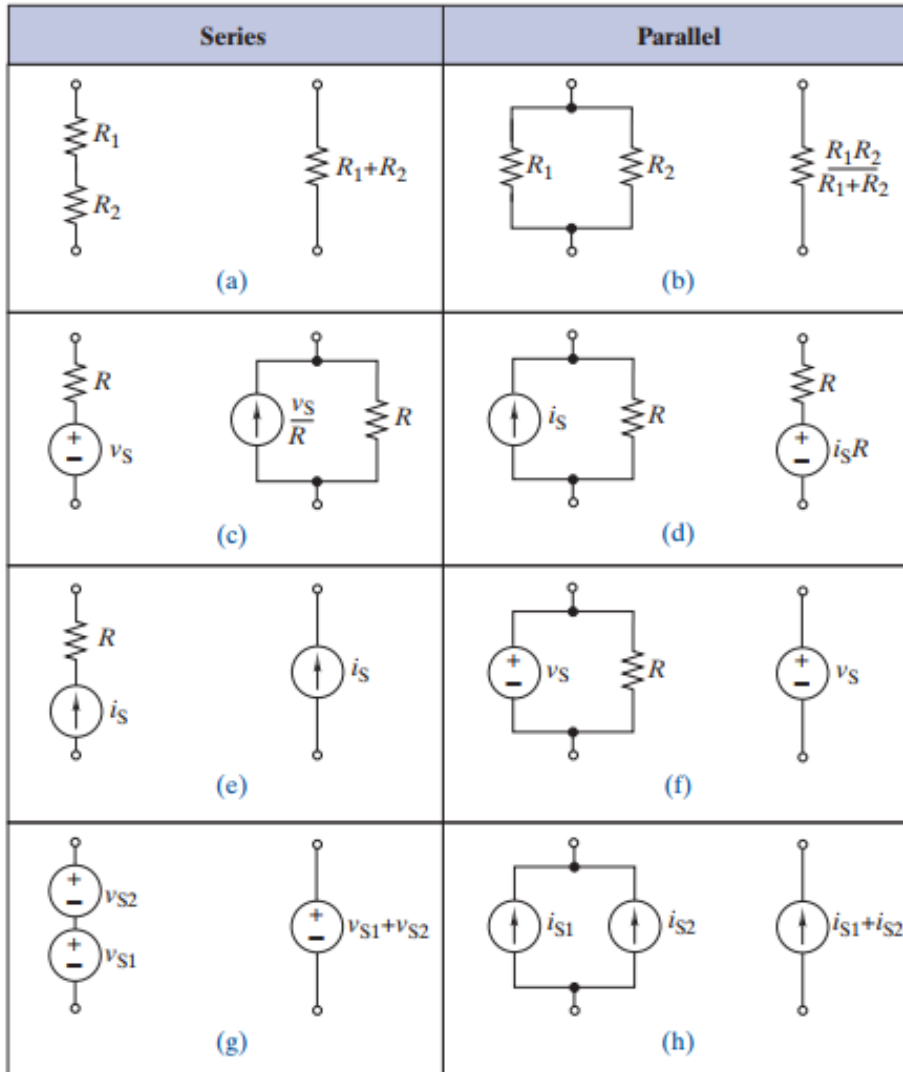


FIGURE 2-37 Summary of two-terminal equivalent circuits.

Consider one resistor in series/parallel with one capacitor?

- In series:  $Z = R + 1/j\omega C$
- In parallel:  $Z = \frac{1}{\frac{1}{R} + j\omega C}$

- Voltage division **RT PP.42**

- In a **series** connection, the voltage across each resistor is equal to its **resistance** divided by the equivalent series **resistance** of the connection times the voltage across the series circuit.

$$v_s = v_1 + v_2 + v_3 \text{ (KVL)}$$

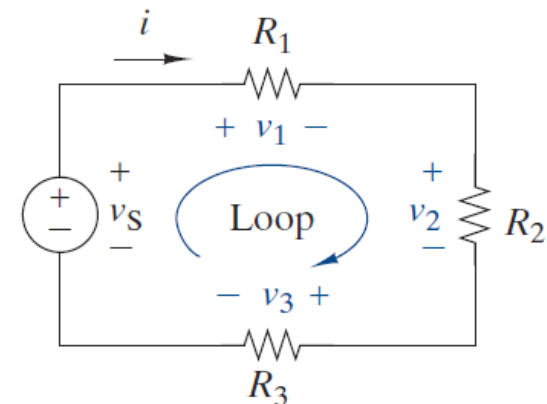
$$v_s = R_1 i + R_2 i + R_3 i \text{ (Ohm's law)}$$

$$i = \frac{v_s}{R_1 + R_2 + R_3}$$

$$\text{thus } v_1 = R_1 i = \left( \frac{R_1}{R_1 + R_2 + R_3} \right) v_s$$

$$v_2 = R_2 i = \left( \frac{R_2}{R_1 + R_2 + R_3} \right) v_s$$

$$v_{13} = R_3 i = \left( \frac{R_3}{R_1 + R_2 + R_3} \right) v_s$$



**FIGURE 2-38** A voltage divider circuit.

- Current division **RT PP.42**

- In a **parallel** connection, the current through each resistor is equal to its **conductance** divided by the equivalent parallel **conductance** of the connection times the current through the parallel circuit.

$$i_s = i_1 + i_2 + i_3 \text{ (KCL)}$$

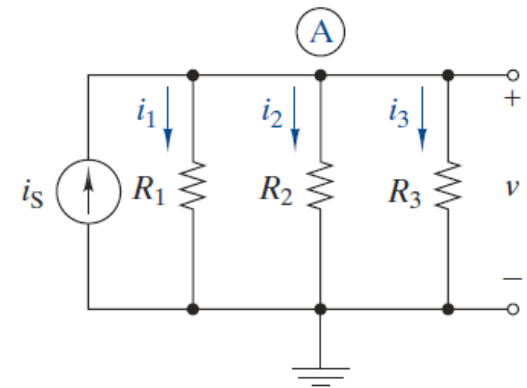
$$i_s = v/R_1 + v/R_2 + v/R_3 \text{ (Ohm's law)}$$

$$v = i_s \frac{1}{1/R_1 + 1/R_2 + 1/R_3}$$

$$\text{thus } i_1 = v/R_1 = \left( \frac{1/R_1}{1/R_1 + 1/R_2 + 1/R_3} \right) i_s$$

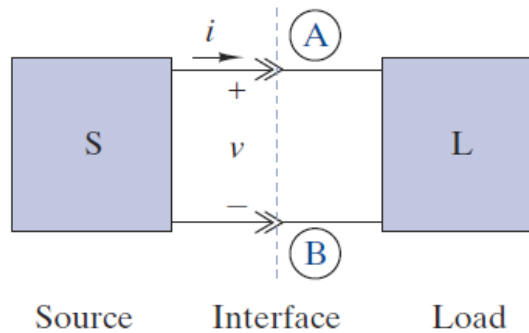
$$i_2 = v/R_2 = \left( \frac{1/R_2}{1/R_1 + 1/R_2 + 1/R_3} \right) i_s$$

$$i_3 = v/R_3 = \left( \frac{1/R_3}{1/R_1 + 1/R_2 + 1/R_3} \right) i_s$$

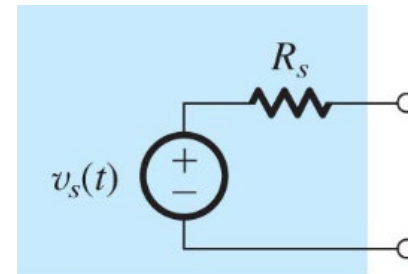


**FIGURE 2-44** A current divider circuit.

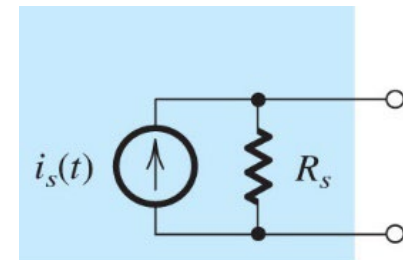
- Thévenin and Norton Equivalent Circuits
  - If the source circuit in a **two-terminal** interface is **linear**, the interface signals  **$v$  and  $i$  don't change** when the source circuit is **replaced by its T/N equivalent circuit**
- RT PP. 107



**FIGURE 3-41** *A two-terminal interface.*



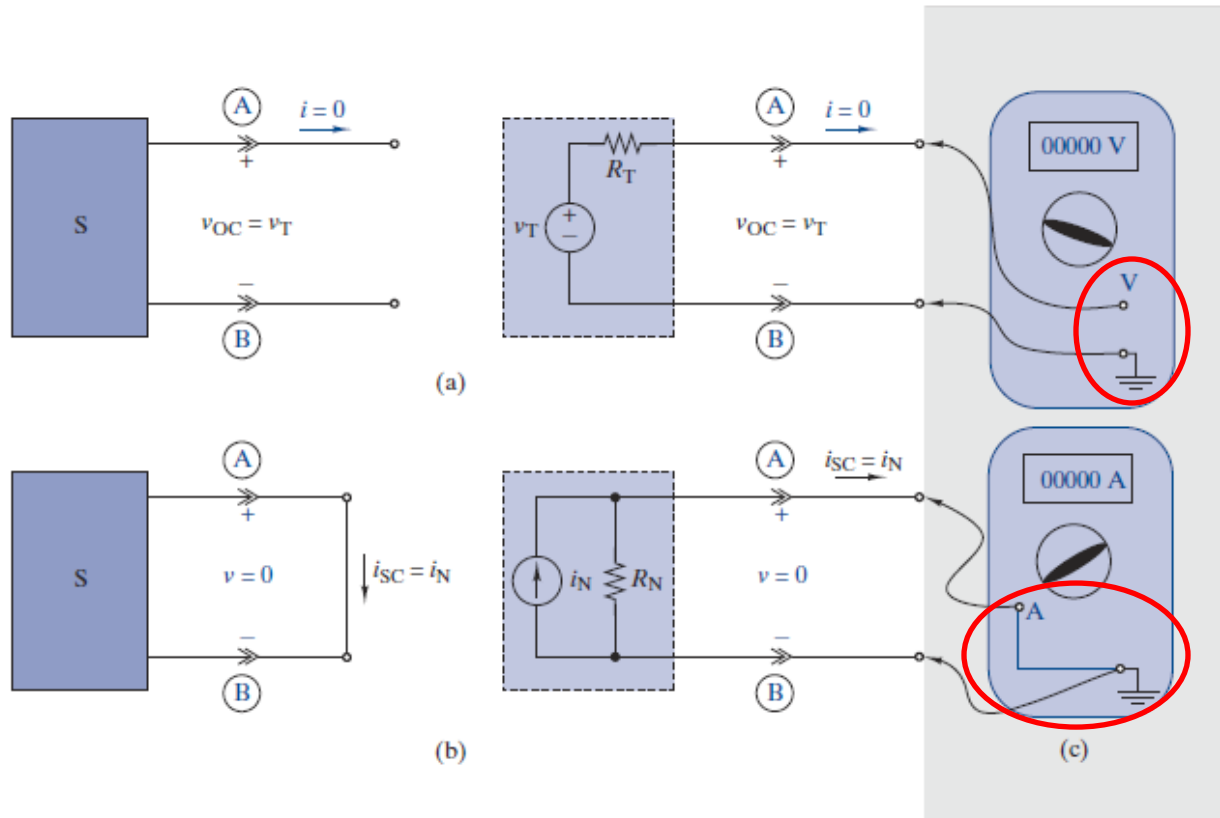
(a)



(b)

**Figure 1.1** Two alternative representations of a signal source: (a) the Thévenin form; (b) the Norton form.

- How to find  $v_T/i_N$  and R?
  - Open circuit for Thévenin  $v_T$
  - Short circuit for Norton  $i_N$



**FIGURE 3-43** Loads used to find Thévenin and Norton equivalent circuits: (a) Open circuit yields the Thévenin voltage. (b) Short circuit yields the Norton current. (c) Measuring  $v_{OC}$  and  $i_{SC}$  using a DMM.

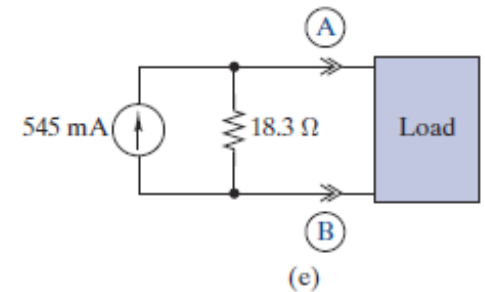
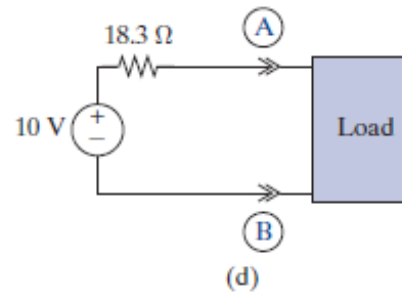
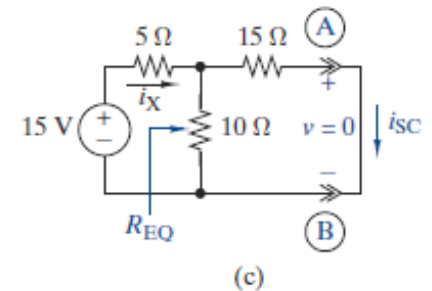
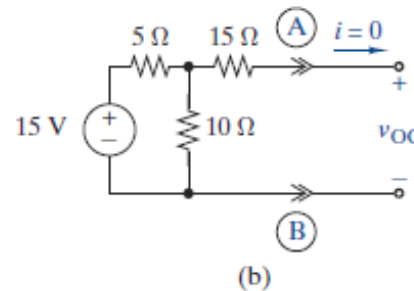
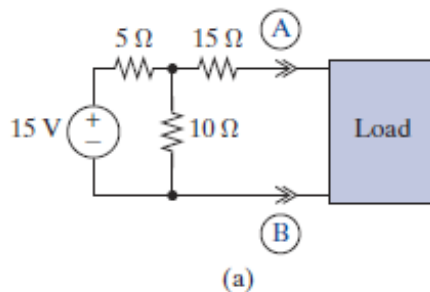
- Find the Thévenin and Norton Equivalent Circuits.

$$v_T = \frac{10}{10+5} \times 15V = 10V \text{ (OPEN LOAD)}$$

$$i_N = \frac{\frac{1}{15}}{\frac{1}{15} + \frac{1}{10} + \frac{1}{5}} \times 3A = 0.5454A \text{ (SHORT LOAD)}$$

$$R = \frac{v_T}{i_N} = 18.3\Omega$$

- Lookback resistance:
  - Turnoff all the sources (RT PP. 115)
  - Grab terminals and walk into



# 1.4 Amplifiers

- Linear amplifier

- Voltage gain  $A_v = \frac{v_o}{v_I} \left( \frac{V}{V} \right)$

- Current gain  $A_i = \frac{i_o}{i_I} \left( \frac{A}{A} \right)$

- Power gain  $A_p = \frac{v_o i_o}{v_I i_I} \left( \frac{W}{W} \right)$

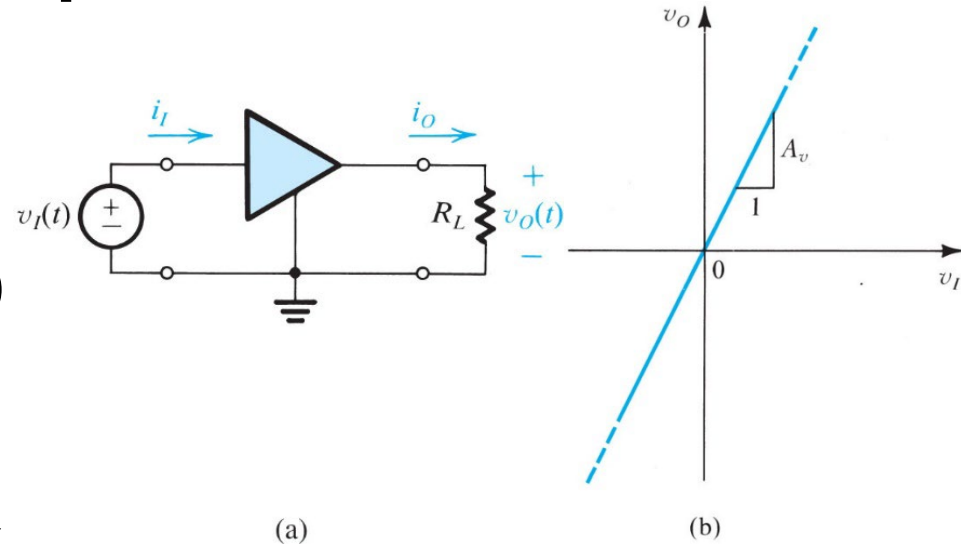


Figure 1.12 (a) an amplifier fed with a signal  $v_I(t)$  (b) Transfer characteristic of a linear voltage amplifier with a voltage gain  $A_v$

- Gain in decibels

- Voltage gain in decibel:  $20 \log A_v \text{ dB}$

- Current gain in decibel:  $20 \log A_i \text{ dB}$

- Power gain in decibel:  $10 \log A_p \text{ dB}$

- Exercise 1.10

- An amplifier has a voltage gain of 100V/V, and a current gain of 1000A/A. Express the voltage and current gains in decibels and find the power gain

(a) voltage gain =  $20 \log 100 = 40\text{db}$

(b) current gain =  $20 \log 1000 = 60\text{db}$

(c) power gain =  $10 \log A_P = 10 \log(A_V A_I) = 50\text{db}$

# 1.5 Circuit Models for Amplifiers

- Voltage amplifier – voltage-controlled ( $v_i, A_{vo}$ )

Open loop gain ( $A_{vo}$ ) = the gain of the component all by itself, with all other components removed. Ideally, **infinite gain can be obtained by an ideal amplifier!**

Closed loop gain ( $A_v$ ) = the gain of the amplifier circuit as a whole, from circuit-input to circuit-output, with all components intact.

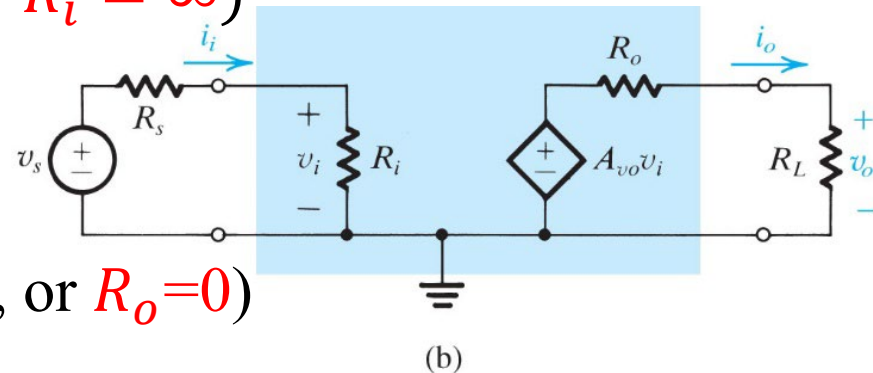
General gain ( $G_v$ ) = the gain of the amplifier from voltage source to load.

$$v_i = v_s \times \frac{R_i}{R_i + R_s} \quad (R_i \gg R_s, \text{ or } R_i = \infty)$$

$$v_o = A_{vo} v_i \times \frac{R_L}{R_L + R_o}$$

$$A_v = \frac{v_o}{v_i} = A_{vo} \frac{R_L}{R_L + R_o} \quad (R_L \gg R_o, \text{ or } R_o = 0)$$

$$G_v = \frac{v_o}{v_s} = A_{vo} \frac{R_L}{R_L + R_o} \frac{R_i}{R_i + R_s}$$



**Figure 1.16 (b)** The voltage amplifier with input signal source and load.



# • The four amplifier models

**Table 1.1** The Four Amplifier Types

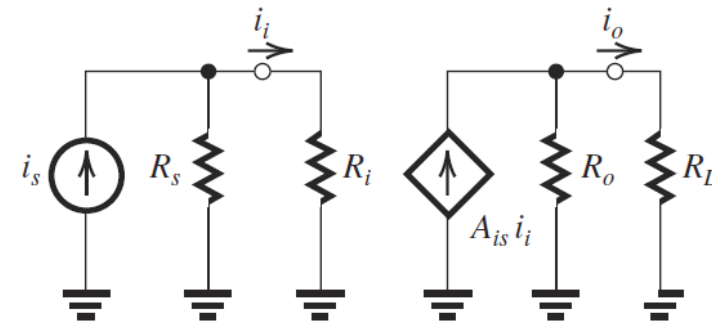
Type	Circuit Model	Gain Parameter	Ideal Characteristics
Voltage Amplifier		Open-Circuit Voltage Gain $A_{vo} \equiv \left. \frac{v_o}{v_i} \right _{i_o=0} \quad (\text{V/V})$	$R_i = \infty$ $R_o = 0$
Current Amplifier		Short-Circuit Current Gain $A_{is} \equiv \left. \frac{i_o}{i_i} \right _{v_o=0} \quad (\text{A/A})$	$R_i = 0$ $R_o = \infty$
Transconductance Amplifier		Short-Circuit Transconductance $G_m \equiv \left. \frac{i_o}{v_i} \right _{v_o=0} \quad (\text{A/V})$	$R_i = \infty$ $R_o = \infty$
Transresistance Amplifier		Open-Circuit Transresistance $R_m \equiv \left. \frac{v_o}{i_i} \right _{i_o=0} \quad (\text{V/A})$	$R_i = 0$ $R_o = 0$

- **Exercise 1.18**

Consider  $\frac{i_o}{i_s}$  of a **current-controlled current amplifier**. Let the amplifier be fed with a current-source  $i_s$  having a resistance  $R_s$ , and let the output be connected to a load resistance  $R_L$ .

$$i_i = \frac{R_s}{R_s + R_i} i_s \quad i_o = \frac{R_o}{R_o + R_L} A_{is} i_i$$

$$\text{Thus, } \frac{i_o}{i_s} = A_{is} \frac{R_o}{R_o + R_L} \frac{R_s}{R_s + R_i}$$

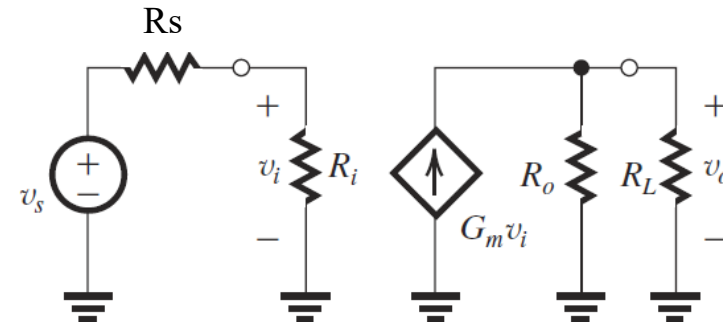


- **Exercise 1.19**

Consider  $\frac{v_o}{v_s}$  of a transconductance amplifier (**voltage-controlled current amplifier**). Let the voltage-source  $v_s$  having a resistance  $R_s$ , and the output be connected to a load resistance  $R_L$ .

$$v_i = \frac{R_i}{R_s + R_i} v_s \quad v_o = G_m v_i (R_o \parallel R_L)$$

$$\text{Thus, } \frac{v_o}{v_s} = G_m \frac{R_i}{R_s + R_i} (R_o \parallel R_L)$$

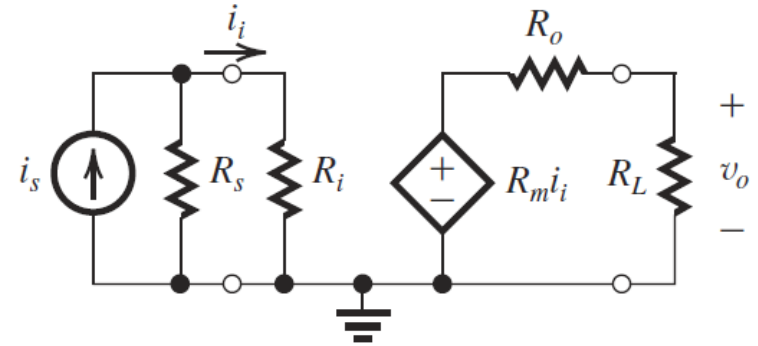


## • Exercise 1.20

Consider  $\frac{v_o}{i_s}$  of a transresistance amplifier (**current-controlled voltage amplifier**). Let the amplifier be fed with a current-source  $i_s$  having a resistance  $R_s$ , and let the output be connected to a load resistance  $R_L$ .

$$i_i = \frac{R_s}{R_s + R_i} i_s \quad v_o = \frac{R_L}{R_L + R_o} R_m i_i$$

$$\text{Thus, } \frac{v_o}{i_s} = R_m \frac{R_L}{R_L + R_o} \frac{R_s}{R_s + R_i}$$

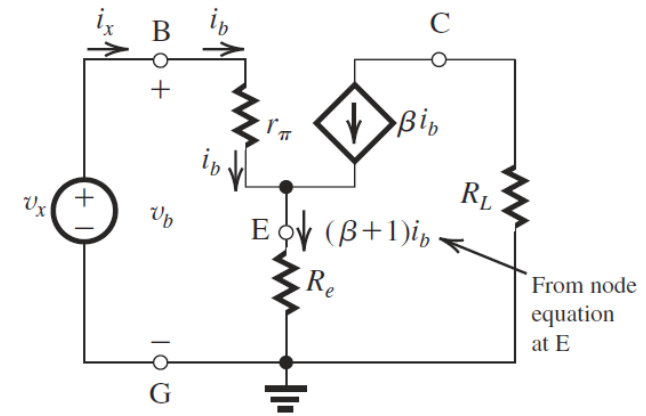


## • Exercise 1.21

Find the input resistance between B and G. The voltage  $v_x$  is a test voltage with the input resistance  $R_{in}$  defined as  $R_{in} = v_x / i_x$ .

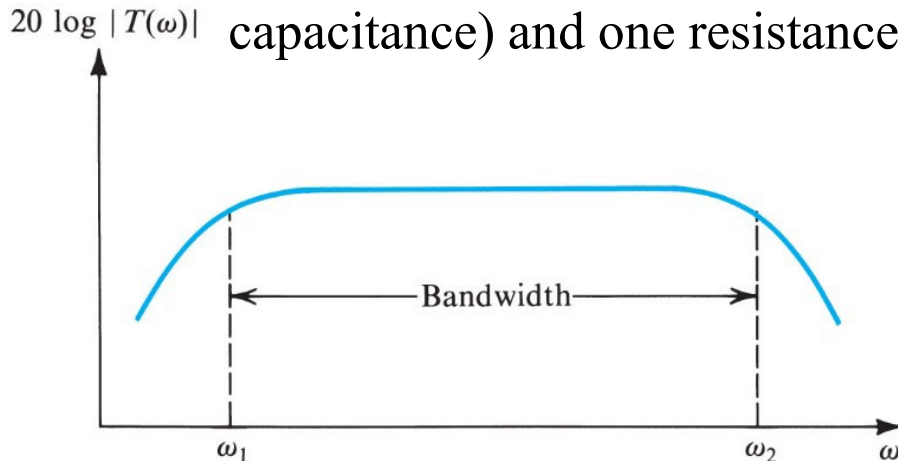
$$v_x = v_b = i_b r_\pi + (\beta + 1) i_b R_e \quad i_x = i_b$$

$$\text{Thus, } R_{in} = \frac{v_x}{i_x} = r_\pi + (\beta + 1) R_e$$



# 1.6 Frequency Response of Amplifier

- Frequency response: (inductance, capacitance)
  - $T(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$
  - gain magnitude  $|T(j\omega)| = \left| \frac{V_o(j\omega)}{V_i(j\omega)} \right|$  in V/V,  $20\lg|T(j\omega)|$  in dB
- Low pass filter, high pass filter: the band of frequencies over which the gain is almost constant, to within 3dB.
- Single-time-constant (STC) network
  - Contains, or can be reduced to, one reactive component (inductance or capacitance) and one resistance



**Figure 1.21** Typical magnitude response of an amplifier:  $|T(\omega)|$  is the magnitude of the amplifier transfer function—that is, the ratio of the output  $V_o(\omega)$  to the input  $V_i(\omega)$ .

- Frequency response – low pass filter

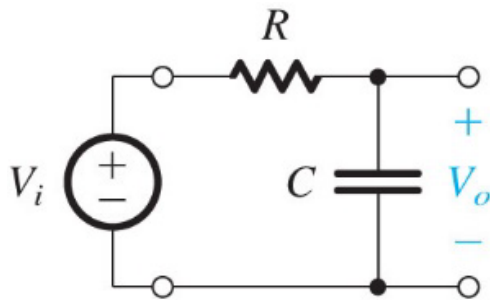
- $T(j\omega) = \frac{1}{1+j\frac{\omega}{1/CR}} = \frac{1}{1+j\frac{\omega}{\omega_0}}$

- **Time constant:  $\tau = CR$ , 3dB/cutoff/corner frequency:  $\omega_0 = 1/\tau$**

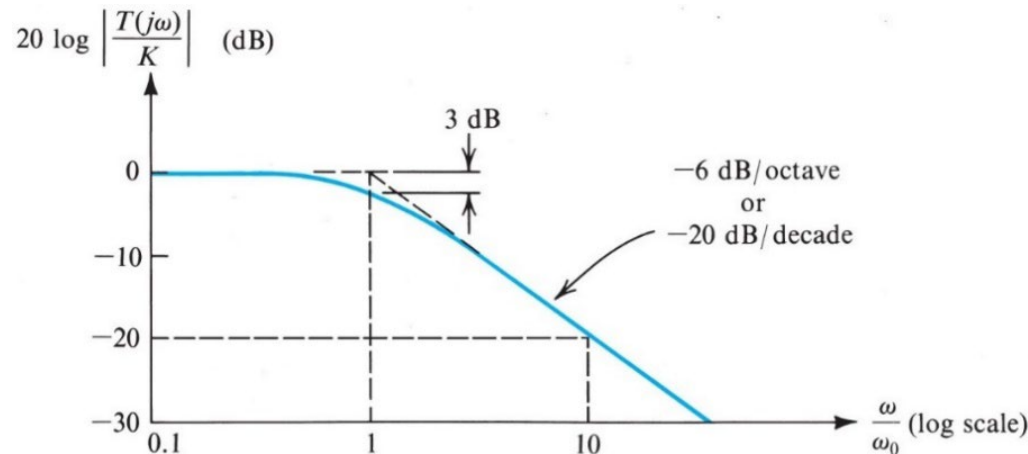
- $|T(j\omega)| = \frac{1}{\sqrt{1+\omega^2/\omega_0^2}}$ , then  **$20 \log(|T(j\omega)/K|)$  is d.c. gain**, in this case

K is 1.

- $\omega = 0, 0 \text{ dB}$
- $\omega = \omega_0, -3 \text{ dB}$
- $\omega = 10\omega_0, -20 \text{ dB}$
- $\omega = 100\omega_0, -40 \text{ dB}$



(a)



(a)

**Figure 1.23 (a)** Magnitude response of STC networks of the low-pass type.

**Figure 1.22** Examples of STC networks: (a) a low-pass filter

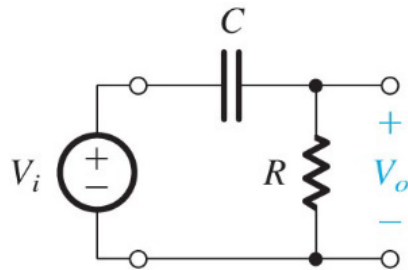
- Frequency response – high pass filter

- $T(j\omega) = \frac{1}{1-j\frac{1/CR}{\omega}} = \frac{1}{1-j\frac{\omega_0}{\omega}}$

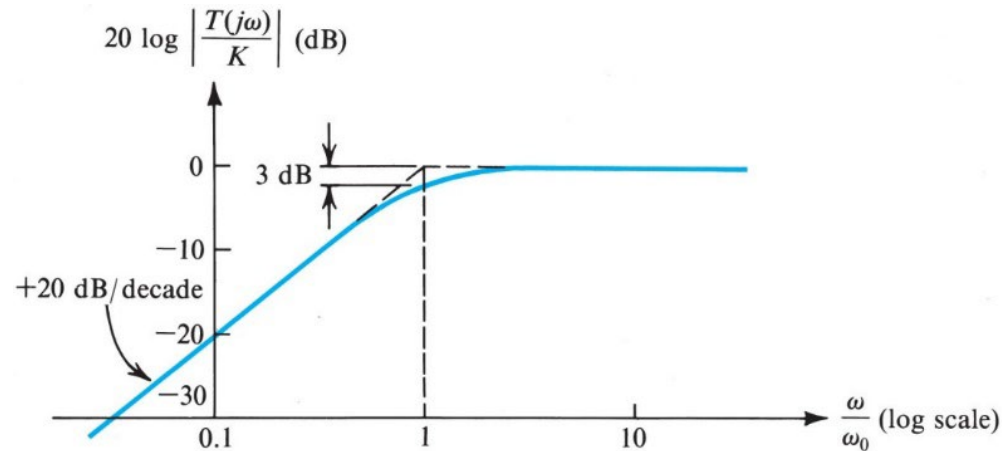
- **Time constant:  $\tau = CR$ , 3dB/cutoff/corner frequency:  $\omega_0 = 1/\tau$**

- $|T(j\omega)| = \frac{1}{\sqrt{1+\omega_0^2/\omega^2}}$ , then  **$20 \log(|T(j\omega)/K|)$  is d.c. gain ( $K=1$ )**

- $\omega = \infty, 0 \text{ dB}$
- $\omega = \omega_0, -3 \text{ dB}$
- $\omega = 0.1\omega_0, -20 \text{ dB}$
- $\omega = 0.01\omega_0, -40 \text{ dB}$



(b)



(a)

**Figure 1.24 (a)** Magnitude response of STC networks of the high-pass type.

**Figure 1.22** Examples of STC networks: (b) a high-pass filter.

- Procedure:
  - Find transfer function
    - Matching the function: LP or HP
    - Find the d.c gain:  $K$  (V/V) or  **$20 \log(|T(j\omega)/K|)$  starting at 0 dB**
    - Find the 3dB **frequency**:  $\omega_0 = 1/\tau$  in rads/s
  - Draw the magnitude response graph (the slop will be 20dB/decade)

**Table 1.2** Frequency Response of STC Networks

	Low-Pass (LP)	High-Pass (HP)
Transfer Function $T(s)$	$\frac{K}{1 + (s/\omega_0)}$	$\frac{Ks}{s + \omega_0}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\frac{K}{1 + j(\omega/\omega_0)}$	$\frac{K}{1 - j(\omega_0/\omega)}$
Magnitude Response $ T(j\omega) $	$\frac{ K }{\sqrt{1 + (\omega/\omega_0)^2}}$	$\frac{ K }{\sqrt{1 + (\omega_0/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega = 0$ (dc)	$K$	$0$
Transmission at $\omega = \infty$	$0$	$K$
3-dB Frequency	$\omega_0 = 1/\tau$ ; $\tau \equiv$ time constant $\tau = CR$ or $L/R$	
Bode Plots	in Fig. 1.23	in Fig. 1.24

Alternative way to find time constant  $\tau$ :

- 1) Set voltage or current source to zero;
- 2) Grab hold two terminals of the capacitor or inductor
- 3) Equivalent resistance  $R$  between these two terminals. Then,  $\tau = \frac{L}{R}$ , or  $\tau = CR$

# • Example 1.5

- Input resistance ( $R_i$ ), capacitance ( $C_i$ ), gain factor ( $\mu$ ), output resistance ( $R_o$ ). Voltage source ( $V_s$  and  $R_s$ ), load resistance ( $R_L$ )

(a) Find voltage gain ( $\frac{V_o}{V_s}$ ) as a function of frequency. From this find expression for the dc gain and the 3-dB frequency.

$$Z_i = R_i \parallel \frac{1}{j\omega C_i} = \frac{R_i}{1 + j\omega C_i R_i}$$

$$\frac{V_i}{V_s} = \frac{Z_i}{Z_i + R_s} = \frac{\frac{R_i}{1 + j\omega C_i R_i}}{\frac{R_i}{1 + j\omega C_i R_i} + R_s} = \frac{R_i}{R_s + R_i} \frac{1}{1 + j\omega \frac{R_s + R_i}{C_i R_s R_i}}$$

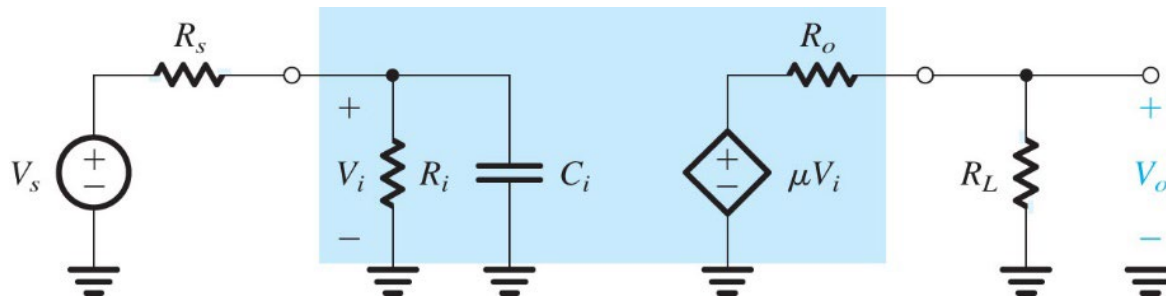


Figure 1.25 Circuit for Example 1.5.

- Example 1.5 Cont.

At the load side, we have  $V_o = \mu V_i \frac{R_L}{R_L + R_o}$ , so

$$\frac{V_o}{V_s} = \mu \frac{R_L}{R_L + R_o} \frac{\frac{R_i}{R_s + R_i}}{1 + j\omega C_i \frac{R_s R_i}{R_s + R_i}} = \mu \frac{R_L}{R_L + R_o} \frac{R_i}{R_s + R_i} \frac{1}{1 + j\omega \frac{R_s + R_i}{C_i R_s R_i}}$$

(Standard form for a low-pass filter/STC network)

Alternative way: Find equivalent resistance as  $R_s // R_i$ ,  $\tau = C_i (R_s // R_i)$

The DC gain is  $\frac{V_o}{V_s} (w = 0) = \mu \frac{R_L}{R_L + R_o} \frac{R_i}{R_s + R_i}$

The 3-dB frequency  $\omega_0 = 1/\tau = 1/C_i (R_s // R_i)$

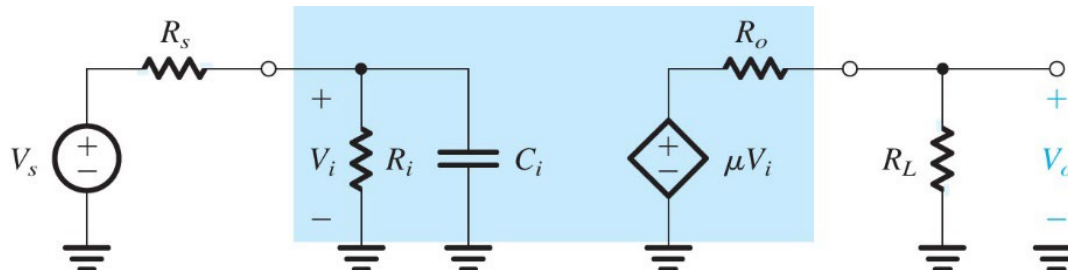


Figure 1.25 Circuit for Example 1.5.

- Example 1.5 Cont.

(b) Find the values of the **dc gain**, the **3-dB frequency**, and the frequency at which the gain becomes 0dB for the case  $R_S = 20K\Omega$ ,  $R_i = 100K\Omega$ ,  $C_i = 60pF$ ,  $\mu = 144$ ,  $R_o = 200\Omega$ ,  $R_L = 1K\Omega$

$$\frac{V_o}{V_s} = \mu \frac{R_L}{R_L + R_o} \frac{R_i}{R_S + R_i} \frac{1}{1 + j\omega / \frac{R_S + R_i}{C_i R_S R_i}}$$

(i)  $k = \frac{V_o}{V_s} (\omega = 0) = 144 \times \frac{1}{1+0.2} \times \frac{100}{100+20} = 100 \text{ V/V (or 40dB)}$

(ii)  $\omega_0 = \frac{20K\Omega + 100K\Omega}{60pF \times 20K\Omega \times 100K\Omega} = \frac{(20+100) \times 10^3}{60 \times 10^{-12} \times 20 \times 100 \times 10^6} = 10^6 \text{ rad/s}$

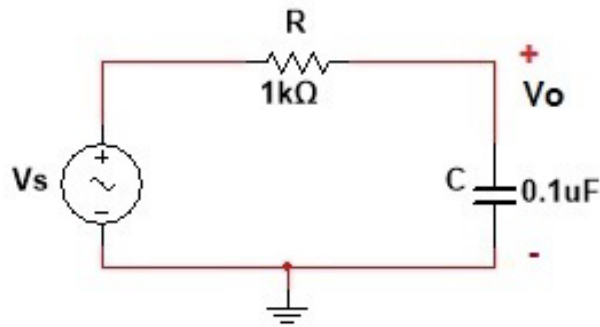
Thus,  $f_0 = 10^6 / 2\pi = 159.2 \text{ kHz}$

(iii) Since the gain falls off at the rate of -20dB/decade, the gain will reach 0dB in 2 decades, thus, unity gain frequency =

$$100\omega_0 = \frac{10^8 \text{ rad}}{s} \text{ or } 15.92 \text{ MHz}$$

# • Experiment 1

- 1) Do frequency sweep till change in  $A_V$  start to occur (using both oscilloscope and bode plotter)
- 2) At what frequency ( $\omega_0 = 2\pi f_0$ ) does  $A_V = 0.707$  (or -3db)



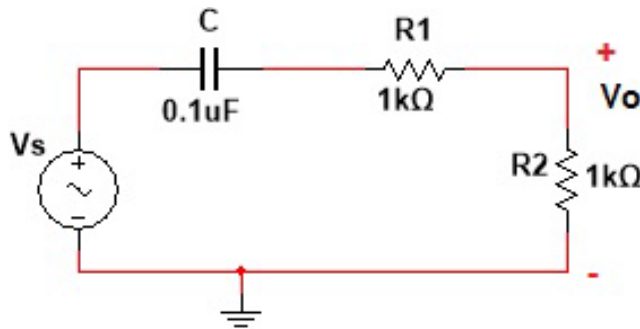
**Table 1.2** Frequency Response of STC Networks

	Low-Pass (LP)	High-Pass (HP)
Transfer Function $T(s)$	$\frac{K}{1 + (s/\omega_0)}$	$\frac{Ks}{s + \omega_0}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\frac{K}{1 + j(\omega/\omega_0)}$	$\frac{K}{1 - j(\omega_0/\omega)}$
Magnitude Response $ T(j\omega) $	$\frac{ K }{\sqrt{1 + (\omega/\omega_0)^2}}$	$\frac{ K }{\sqrt{1 + (\omega_0/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega = 0$ (dc)	$K$	$0$
Transmission at $\omega = \infty$	$0$	$K$
3-dB Frequency	$\omega_0 = 1/\tau$ ; $\tau \equiv$ time constant $\tau = CR$ or $L/R$	
Bode Plots	in Fig. 1.23	in Fig. 1.24

Frequency	100Hz	300Hz	1kHz	?	10kHz	100kHz	1MHz
$V_{o(P-P)}$							
$A_V \left(\frac{V}{V}\right)$							
$A_V (db)$							

# • Experiment 2

- 1) Do frequency sweep till change in  $A_V$  start to occur (using both oscilloscope and bode plotter)
- 2) At what frequency does  $A_V = 0.5 \times 0.707$  (or  $-6db - 3db = -9db$ )



**Table 1.2** Frequency Response of STC Networks

	Low-Pass (LP)	High-Pass (HP)
Transfer Function $T(s)$	$\frac{K}{1 + (s/\omega_0)}$	$\frac{Ks}{s + \omega_0}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\frac{K}{1 + j(\omega/\omega_0)}$	$\frac{K}{1 - j(\omega_0/\omega)}$
Magnitude Response $ T(j\omega) $	$\frac{ K }{\sqrt{1 + (\omega/\omega_0)^2}}$	$\frac{ K }{\sqrt{1 + (\omega_0/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega = 0$ (dc)	$K$	$0$
Transmission at $\omega = \infty$	$0$	$K$
3-dB Frequency	$\omega_0 = 1/\tau; \tau \equiv$ time constant $\tau = CR$ or $L/R$	
Bode Plots	in Fig. 1.23	in Fig. 1.24

Frequency	100Hz	300Hz	1kHz	?	10kHz	100kHz	1MHz
$V_{o(P-P)}$							
$A_V \left(\frac{V}{V}\right)$							
$A_V (db)$							

# HW1

- Problems:
  - PP.47, 1.19 – Circuits analysis
  - PP.48, 1.21 – AC circuits
  - PP.50, 1.43 – Circuit models for amplifiers
  - PP.54, 1.68 – Frequency response of amplifiers
- Submission requirement:
  - Add the cover page!!!
  - [Print the HW1.pdf out and answer all the questions \(download on the blackboard\)](#)